

Cutoff Wavenumbers and Modes for Annular-Cross-Section Waveguide with Eccentric Inner Conductor of Small Radius

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Abstract — Analytical expressions are derived for the cutoff wavenumbers and the corresponding modes in annular-cross-section waveguides having inner conductors of small radius. Waveguides with circular and rectangular outer boundary are considered. In the case of the circular eccentric annular waveguide, comparison is made between the values of cutoff wavenumbers computed from the expressions derived in this paper and data obtained by a more rigorous numerical technique.

I. INTRODUCTION

COMPUTATION OF cutoff wavenumbers for uniform waveguides with eccentric annular cross section has been the subject of numerous investigations [1]–[4], [9], [10].¹ The various techniques suggested for that purpose are for the most part rigorous in nature and require considerable numerical analysis. A relatively important limiting case which has not received much attention, and yet is of practical interest, is the annular waveguide with small ratio of inner to outer conductor dimensions. Configurations of this type arise, for example, in the analysis of cavities excited by thin probes [5].

The purpose of this paper is to derive approximate analytical expressions for the cutoff wavenumbers and the corresponding modal wave functions for annular waveguides with small ratios of inner to outer conductor dimensions. Derivation of the results is based to a large extent on the perturbation technique outlined in the investigation entitled “Distortion of Standing Wave by a Strip,” found in reference [6].

Circular waveguide with eccentrically located inner conductor of small radius is analyzed in Section II. Cutoff wavenumbers computed therein are compared with data numerically generated from a more rigorous formulation [1].

Section III considers the case of rectangular waveguide with arbitrarily located small inner conductor. Again, the cutoff wavenumbers and the corresponding modal wave functions are derived.

Manuscript received October 16, 1986; revised January 4, 1987. This work is an extension of that sponsored by RADC under Contract F19628-83K-0052 and by NASA under Grant NAG 4-418.

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IEEE Log Number 8613828.

¹Only a partial list of relevant publications is given in this paper.

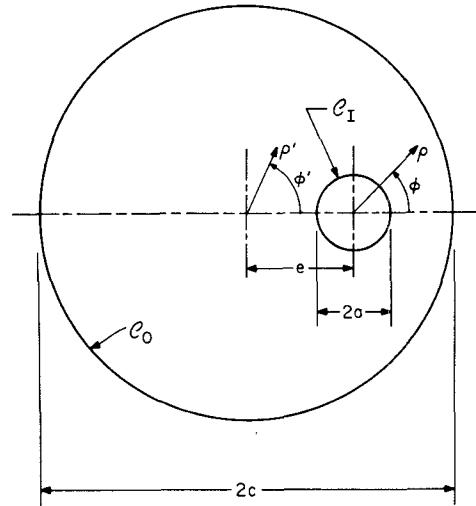


Fig. 1. Cross section of an annular waveguide with circular outer conductor.

II. CIRCULAR WAVEGUIDE WITH AN ECCENTRIC INNER CONDUCTOR OF SMALL RADIUS

Approximate formulas for the cutoff wavenumbers and the corresponding modes in a circular waveguide with eccentric inner conductor of small radius are derived in this section. A cross section of the waveguide, with the relevant dimensions indicated, is shown in Fig. 1. Analysis of the TM modes symmetric with respect to ϕ' is presented in detail. The antisymmetric TM modes are not considered, since they are insignificantly perturbed by the small inner conductors for which the theory presented here is valid. The results for the TE case can be obtained in an analogous manner.

The TM modes in a uniform waveguide can be derived from a scalar potential function ψ_{vm} satisfying the wave equation

$$(\nabla_t^2 + k_{vm}^2) \psi_{vm} = 0 \quad (1)$$

and subject to the homogeneous Dirichlet boundary condition [7]. In the absence of the inner conductor in a circular waveguide of radius c , the solution of (1) is known to be

$$\psi_{vm} = J_m(k_{vm}\rho') \cos(m\phi'), \quad J_m(k_{vm}c) = 0 \quad (2)$$

where J_m denotes a Bessel function of the first kind of order m . It is intuitively clear that the presence of a small

inner conductor slightly perturbs the above solution. Derivation of the modified solution, denoted by Ψ_{vm} , is facilitated by the introduction of several hypotheses, detailed discussion of which can be found in [6]. A brief restatement thereof follows.

Initially, it is assumed that the standing wave ψ_{vm} , which is expressible as a combination of diverging and converging cylindrical waves, is incident upon the inner conductor and is scattered by it. The effect of the scatterer on the incident waves is computed as though these waves were out in the unbounded medium, i.e., with the outer wall boundary C_O at infinity. It is hypothesized that the value of Ψ_{vm} at the inner conductor boundary C_I should not be very sensitive to the presence or absence of the outer wall, so long as the inner conductor is small and is not situated close to C_O . Consequently, the behavior of Ψ_{vm} on C_I is known fairly accurately from the analysis performed with C_O at infinity. The behavior of the modified wavefunction Ψ_{vm} in the rest of the waveguide cross section can be found with the aid of the following equa-

$$\Psi_{vm}(\rho, \phi) = \sum_{p=0}^{\infty} E(m, p, e) \left[J_p(k_{vm}\rho) - \frac{J_p(k_{vm}a)}{H_p^{(2)}(k_{vm}a)} \cdot H_p^{(2)}(k_{vm}a) \right] \cdot \cos(p\phi) \quad (7)$$

$$\frac{\partial \Psi_{vm}}{\partial n} \Big|_{C_I} = - \frac{\partial \Psi_{vm}(\rho, \phi)}{\partial \rho} \Big|_{\rho=a} = - \frac{2j}{\pi a} \sum_{p=0}^{\infty} E(m, p, e) \frac{1}{H_p^{(2)}(k_{vm}a)} \cdot \cos(p\phi) \quad (8)$$

tion:

$$\Psi_{vm}(\rho'_o, \phi'_o) = - \oint_{C_I} \frac{\partial \Psi_{vm}}{\partial n}(\rho'_s, \phi'_s) G(\rho'_o, \phi'_o | \rho'_s, \phi'_s) dc \quad (3)$$

derived using Green's theorem. The subscripts o and s in the above equation refer to the observation and source coordinates, respectively. $G(\rho'_o, \phi'_o | \rho'_s, \phi'_s)$ denotes the scalar Green function satisfying the homogeneous Dirichlet boundary condition on C_O . Application of standard analytical techniques yields the following expression for $G(\rho'_o, \phi'_o | \rho'_s, \phi'_s)$:

$$G(\rho'_o, \phi'_o | \rho'_s, \phi'_s) = \sum_{n=0}^{\infty} \sum_{\xi=1}^{\infty} \frac{J_n(k_{qn}\rho'_o) J_n(k_{\xi n}\rho'_s)}{N_{\xi n}^2(k^2 - k_{\xi n}^2)} \cdot \cos n(\phi'_o - \phi'_s) \quad (4a)$$

where

$$N_{\xi n} = \left\{ \frac{\pi c^2}{\epsilon_n} J_{n+1}^2(k_{\xi n}c) \right\}^{1/2} \quad (4b)$$

$$\epsilon_n = \begin{cases} 1 & n=0 \\ 2 & n \neq 0. \end{cases}$$

In accordance with the procedure outlined in the preceding paragraphs, let it be assumed that the combination of cylindrical waves given by ψ_{vm} is incident upon the inner conductor. The conductor is centered at the origin of the coordinate system (ρ, ϕ) , shown in Fig. 1. The addition theorem for Bessel functions [8] is used to transform ψ_{vm} (eq. (2)) from the (ρ', ϕ') into the (ρ, ϕ) coordinate system.

The resulting expression is given by

$$\psi_{vm}(\rho, \phi) = \sum_{p=0}^{\infty} E(m, p, e) J_p(k_{vm}\rho) \cos(p\phi) \quad (5a)$$

where

$$E(m, p, e) = \frac{\epsilon_p}{2} [J_{m-p}(k_{vm}e) + (-1)^p J_{m+p}(k_{vm}e)]. \quad (5b)$$

The scattered field can be expressed as a sum of outgoing cylindrical waves²:

$$\psi^s = \sum_{p=0}^{\infty} A_p H_p^{(2)}(k_{vm}\rho) \cos(p\phi) \quad (6)$$

where $H_p^{(2)}$ is the Hankel function of the second kind of order p . The unknown coefficients A_p are found by enforcing the boundary condition requiring $\Psi_{vm} = \psi_{vm} + \psi^s$ to vanish on $C_I(\rho = a)$. The incident-plus-scattered wave solution and its normal derivative evaluated on C_I are given by

where the Wronskian of Bessel's equation was used in deriving (8). In the limit, as the radius a goes to zero, (8) can be approximated by the following expression:

$$\frac{\partial \Psi_{vm}}{\partial n} \Big|_{C_I} \approx \frac{2}{\pi a} \frac{E(m, 0, e)}{f(k_{vm}a)} = \frac{2}{\pi a} \frac{J_m(k_{vm}e)}{f(k_{vm}a)} \quad (9a)$$

where $f(k_{vm}a)$ is the small argument expansion of $jH_0^{(2)}$, given by [8]

$$f(k_{vm}a) = \frac{2}{\pi} \left[\log \left(\frac{k_{vm}a}{2} \right) + \gamma \right] \quad (9b)$$

and $\gamma = 0.5772$ is Euler's constant.

At this point, (4) and (9) can be substituted into (3) and the prescribed integration performed. The most expedient way to perform the integration is by transforming all the functions involving the variables of integration into the (ρ, ϕ) coordinate system. In the case of the Green function $G(\rho'_o, \phi'_o | \rho'_s, \phi'_s)$, the required transformation is facilitated by the use of the addition theorem for Bessel functions. The transformation is applied to source coordinate (ρ'_s, ϕ'_s) and yields the following expression:

$$G(\rho'_o, \phi'_o | \rho'_s, \phi'_s) \Big|_{C_I} = G(\rho', \phi' | \rho, \phi) \Big|_{\rho=a} = \sum_{n=0}^{\infty} \sum_{\xi=1}^{\infty} \frac{J_n(k_{\xi n}\rho')}{N_{\xi n}^2(k^2 - k_{\xi n}^2)} \cdot \sum_{q=0}^{\infty} E(n, q, e) J_q(k_{\xi n}a) [\cos(n\phi') \cos(q\phi) + \sin(n\phi') \sin(q\phi)] \quad (10)$$

²A $e^{i\omega t}$ time dependence is assumed throughout the paper.

where the subscript o is unambiguously omitted from ρ', ϕ' . Equation (3) can now be rewritten as

$$\Psi_{vm}(\rho', \phi') \approx - \int_0^{2\pi} \left\{ -\frac{\partial \Psi_{vm}}{\partial \rho} \cdot G(\rho', \phi' | \rho, \phi) \right\} |_{\rho=a} \cdot (ad\phi) \quad (11)$$

straightforward evaluation of which yields

$$\Psi_{vm}(\rho', \phi') = - \frac{4J_m(k_{vm}e)}{f(k_{vm}a)} \cdot \sum_{n=0}^{\infty} \sum_{\xi=1}^{\infty} \frac{J_n(k_{\xi n}e) J_n(k_{\xi n}\rho')}{N_{\xi n}^2(k^2 - k_{\xi n}^2)} \cdot J_0(k_{\xi n}a) \cos(n\phi'). \quad (12)$$

As it stands, (12) is not a solution of the eigenvalue problem for the annular domain bound by C_O and C_I until the parameter k is replaced by the correct eigenvalue. Henceforth, this eigenvalue or the cutoff wavenumber for the annular domain will be denoted by K_{vm} . Noting the fact that ψ_{vm} and Ψ_{vm} satisfy the wave equation, with wavenumbers k_{vm} and K_{vm} , respectively, and using Green's theorem, it can be shown that K_{vm} satisfies the following formula [6]:

$$K_{vm}^2 = k_{vm}^2 - \frac{\oint_{C_I} \psi_{vm} \frac{\partial \Psi_{vm}}{\partial n} ds}{\iint_{S_A} \psi_{vm} \Psi_{vm} ds} \quad (13)$$

where S_A is the annular region bounded by C_O and C_I . Using the hypothesis that Ψ_{vm} does not significantly differ from ψ_{vm} over most of S_A , the integral in the denominator of (13) can be approximated in the following manner:

$$\iint_{S_A} \psi_{vm} \Psi_{vm} ds \approx \iint_{S_C} \psi_{vm}^2 ds = N_{vm}^2 \quad (14)$$

where S_C is the circular area bounded by C_O , and N_{vm} is given by (4b). Substitution of (5a) and (9) into (13) yields the following result:

$$K_{vm}^2 \approx k_{vm}^2 - \frac{4J_m^2(k_{vm}e) J_0(k_{vm}a)}{N_{vm}^2 f(k_{vm}a)}. \quad (15)$$

Note that the first term in the above expression for K_{vm}^2 is just the square of the original cutoff wavenumber, whereas the second term constitutes a small perturbation thereof.

To complete the derivation of Ψ_{vm} , a substitution of K_{vm} for k in (12) is made, and the result simplified using the fact that

$$K_{vm}^2 - k_{\xi n}^2 \approx \begin{cases} k_{vm}^2 - k_{\xi n}^2 & \xi n \neq vm \\ -\frac{4J_m^2(K_{vm}e) J_0(k_{vm}a)}{N_{vm}^2 f(k_{vm}a)} & \xi n = vm. \end{cases} \quad (16)$$

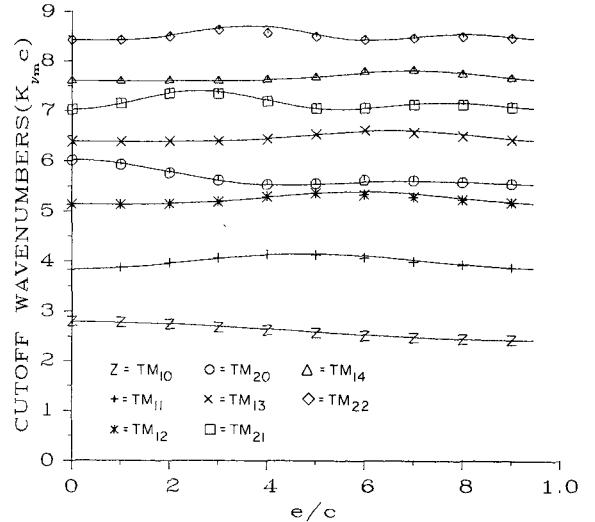


Fig. 2. Cutoff wavenumbers for the TM modes in a waveguide with a PEC circular outer conductor and an eccentrically located PEC inner conductor, $a/c = 0.01$. (Markers are placed at points computed using the rigorous technique found in [1].)

The final expression for Ψ_{vm} can be written as follows:

$$\Psi_{vm}(\rho', \phi') \approx \psi_{vm}(\rho', \phi') - \frac{4J_m(k_{vm}e)}{f(k_{vm}a)} \cdot \sum_{n=0}^{\infty} \sum_{\xi=1}^{\infty} \frac{J_n(k_{\xi n}e) J_n(k_{\xi n}\rho')}{N_{\xi n}^2(k_{vm}^2 - k_{\xi n}^2)} \cdot J_0(k_{\xi n}a) \cos(n\phi') \quad (17)$$

where in addition to the original wave function ψ_{vm} there is a small term representing the perturbation by the inner conductor.

To verify the accuracy of the presented solution, a comparison is made between the results computed from (15) and those obtained by the more rigorous technique of reference [1]. The eight lowest cutoff wavenumbers are plotted in Figs. 2 and 3 as functions of the eccentricity e/c for a/c ratios of .01 and .03, respectively. The solid curves represent the data computed from (15), and the markers are placed at points computed by the alternate method [1]. The agreement between the separately computed sets of data appears to be very good for the cases considered.

III. RECTANGULAR WAVEGUIDE WITH A CYLINDRICAL INNER CONDUCTOR OF SMALL RADIUS

Analysis of the circular waveguide with an eccentrically located inner conductor of small radius was presented in the preceding section. It should be noted, however, that the general technique used in Section II to derive the cutoff wavenumbers and the corresponding modes is applicable to other waveguide cross-section geometries. In this section, it will be used to solve for the TM modes in a rectangular waveguide with inner conductor of small radius. Fig. 4 illustrates the geometry, along with the relevant coordinate systems and dimensions. To avoid repetition

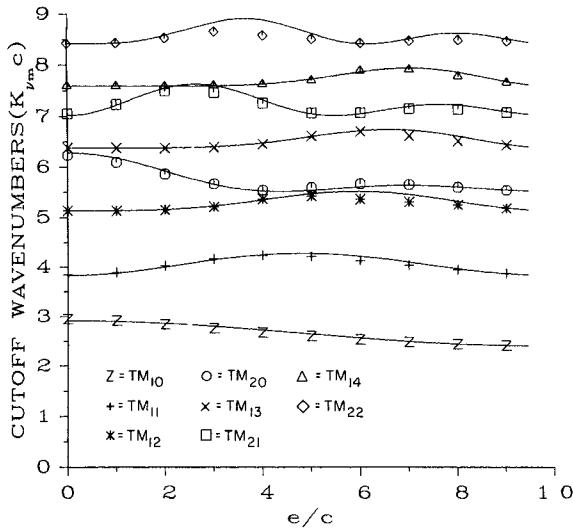


Fig. 3. Cutoff wavenumbers for the TM modes in a waveguide with a PEC circular outer conductor and an eccentrically located PEC inner conductor, $a/c = 0.03$. (Markers are placed at points computed using the rigorous technique found in [1].)

and keep the presentation as brief as possible, all the assumptions and hypotheses stated in the preceding section will be omitted here. Only the most essential steps in the derivation will be shown, since it is completely analogous to the one found in Section II.

In the absence of the inner conductor, the mn th TM mode in the rectangular waveguide can be determined from the scalar potential function:

$$\psi_{mn} = \sin(k_{xm}x') \sin(k_{yn}y') \quad (18)$$

where

$$k_{xm} = \left(\frac{m\pi}{c} \right) \quad k_{yn} = \left(\frac{n\pi}{b} \right).$$

The wave function ψ_{mn} is expressible as a combination of four plane waves. It is assumed that these waves are incident upon the inner conductor. The scattering problem of plane wave by a circular cylinder is well documented [6] and its solution need not be reconsidered here. The total incident-plus-scattered wave $\Psi_{mn} = \psi_{mn} + \psi^s$, satisfying the homogeneous Dirichlet boundary condition on C_I , is found by superposition of the solutions for the four plane waves constituting ψ_{mn} . As in Section II, the quantity of interest is the value of $\partial \Psi_{mn} / \partial n|_{C_I}$. Straightforward analysis shows that for small values of $k_{mn}a$,

$$\frac{\partial \Psi_{mn}}{\partial n} \Big|_{C_I} \approx \frac{2}{\pi a} \cdot \frac{\sin(k_{xm}x_c) \sin(k_{yn}y_c)}{f(k_{mn}a)} \quad (19)$$

where $k_{mn} = \sqrt{k_{xm}^2 + k_{yn}^2}$ and a , x_c , and y_c are as shown in Fig. 4.

The form of the modified wave function Ψ_{mn} at arbitrary points within the guide cross section is found with the aid of (3). For the case of the rectangular waveguide being considered here, the Green function appearing in (3)

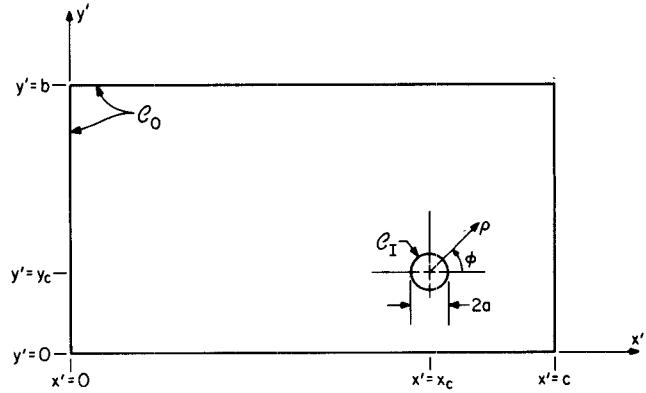


Fig. 4. Cross section of an annular waveguide with rectangular outer conductor.

is given by

$$G(x'_o, y'_o | x'_s, y'_s) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(k_{xp}x'_o) \sin(k_{yq}y'_o) \sin(k_{xp}X'_s) \sin(k_{yq}y'_s)}{N^2(k^2 - k_{pq}^2)} \quad (20a)$$

where

$$N = \frac{\sqrt{bc}}{2}. \quad (20b)$$

The integration prescribed in (3) is facilitated by several transformations of (20). The source coordinates transformation $x'_s = x_c + \rho \cos \phi$, $y'_s = y_c + \rho \sin \phi$ is implemented first. This allows the term $\sin(k_{xp}x'_s) \sin(k_{yq}y'_s)$ in (20a) to be rewritten as follows:

$$\begin{aligned} & \sin(k_{xp}x'_s) \sin(k_{yq}y'_s) \\ &= [\sin(k_{xp}x_c) \cos(k_{xp}\rho \cos \phi) \\ & \quad + \cos(k_{xp}x_c) \sin(k_{xp}\rho \cos \phi)] \\ & \quad \cdot [\sin(k_{yq}y_c) \cos(k_{yq}\rho \sin \phi) \\ & \quad + \cos(k_{yq}y_c) \sin(k_{yq}\rho \sin \phi)]. \end{aligned} \quad (21)$$

Equation (21) is further expanded in a series of terms involving Bessel functions $J_\nu(k_{xp}\rho)$, $J_\nu(k_{yq}\rho)$ and angular harmonics $\cos(\nu\phi)$, $\sin(\nu\phi)$ [8]. The first and only term of the expansion required for subsequent calculations is given by

$$\begin{aligned} & \sin(k_{xp}x'_s) \sin(k_{yq}y'_s) \\ & \approx \sin(k_{xp}x_c) \sin(k_{yq}y_c) J_0(k_{xp}\rho) J_0(k_{yq}\rho). \end{aligned} \quad (22)$$

Substitution of (19) and (20a) into (3) and subsequent use of (22) yield the formal expression for the modified wave function Ψ_{mn} :

$$\begin{aligned} \Psi_{mn}(x', y') & \approx \frac{-4 \sin(k_{xm}X_c) \sin(k_{yn}y_c)}{f(k_{mn}a)} \\ & \quad \cdot \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(k_{xp}x') \sin(k_{yq}y') \sin(k_{xp}x_c)}{N^2(k^2 - k_{pq}^2)} \\ & \quad \cdot \sin(k_{yq}y_c) \cdot J_0(k_{xp}a) J_0(k_{yq}a). \end{aligned} \quad (23)$$

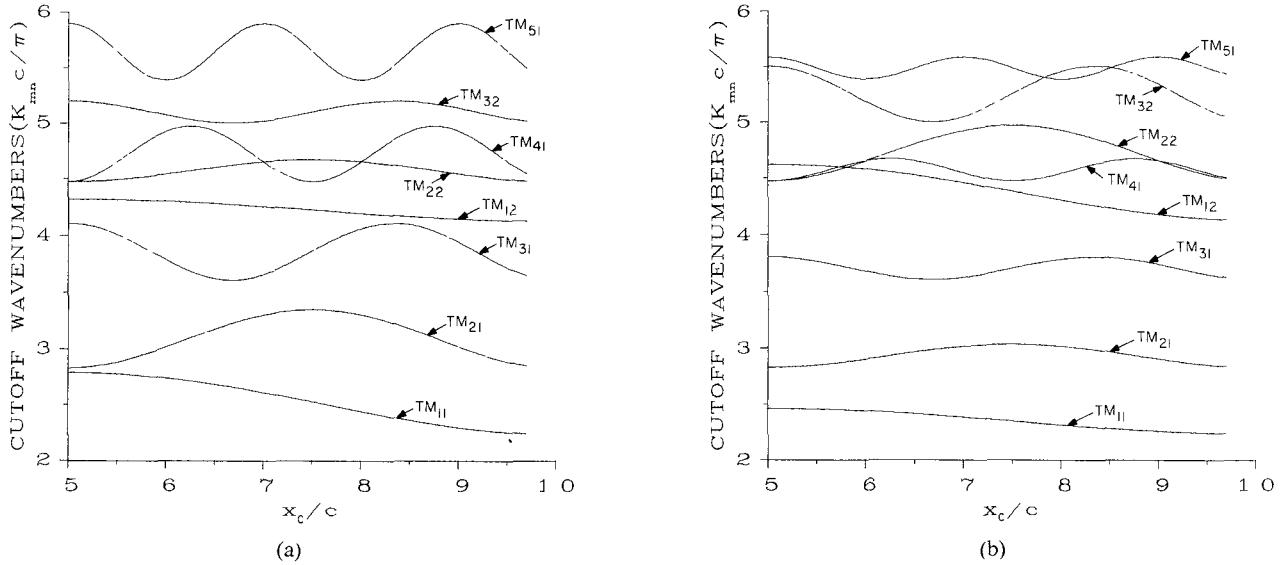


Fig. 5. (a) Cutoff wavenumbers for the TM modes in a waveguide with a PEC rectangular outer conductor and an eccentrically located PEC inner conductor, $b/c = 0.5$, $a/c = 0.03$, $y_c/c = 0.1$. (b) Cutoff wavenumbers for the TM modes in a waveguide with a PEC rectangular outer conductor and an eccentrically located PEC inner conductor, $b/c = 0.5$, $a/c = 0.03$, $y_c/c = 0.2$.

As previously noted in Section II, (23) is not correct until and unless k is replaced by the modified wavenumber K_{mn} . Equation (13) is used to calculate K_{mn} , which after some analysis can be shown to satisfy the following formula:

$$K_{mn}^2 \approx k_{mn}^2 - \frac{4 \sin^2(k_{xm}x_c) \sin^2(k_{yn}y_c)}{N^2 f(k_{mn}a)} \cdot J_0(k_{xm}a) J_0(k_{yn}a). \quad (24)$$

The final form of the modified wave function Ψ_{mn} is obtained when (24) is substituted into (23) and the resulting expression simplified in the manner previously described in (16):

$$\Psi_{mn}(x', y') \approx \psi_{mn}(x', y') - \frac{4 \sin(k_{xm}x_c) \sin(k_{yn}y_c)}{f(k_{mn}a)} \cdot \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(k_{xp}x') \sin(k_{yq}y') \sin(k_{xp}x_c) \sin(k_{yq}y_c)}{N^2 (k_{mn}^2 - k_{pq}^2)} \cdot J_0(k_{xp}a) J_0(k_{yq}a). \quad (25)$$

Fig. 5(a) and (b) illustrates the variation of K_{mn} with x_c for a rectangular waveguide with ratio $b/c = 0.5$. The results are plotted over the domain $0.5 \leq x_c/c \leq 0.97$, $y_c/c = 0.1, 0.2$.

IV. CONCLUSIONS

A perturbation technique was applied to derive analytical expressions for the cutoff wavenumber and the corresponding modes in circular and rectangular waveguides with inner conductors of small radius. The technique is applicable to other separable-cross-section waveguides and

to inner conductor cross sections other than circular. The perturbation procedure is restricted to cases where the ratio of the inner to the outer conductor dimension is much smaller than 1. This restriction can be relaxed somewhat, for the lowest cutoff wavenumbers, if the inner conductor is kept well away from the outer walls, i.e., close to the center.

In the case of the circular waveguide, the results derived herein were compared with those obtained by a more rigorous procedure. There was excellent agreement between the two sets of data for all values of eccentricity (e/c) so long as ratio of inner to outer conductor dimensions was less than 0.05. As this value is increased, the agreement deteriorates, although for small values of e/c

and the lowest cutoff wavenumbers, it still remains quite good.

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